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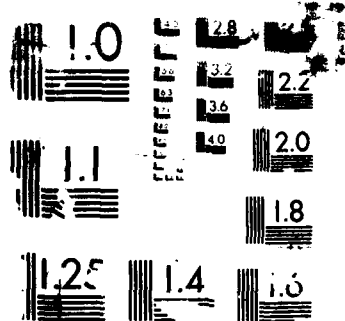# AD-A189 251

A Direct Correlation of

Strength with Impact Velocity

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This paper presents a method for analyzing the results of a Taylor impact test. From post mortem measurements of final specimen length and final undeformed specimen length the dynamic material strength on impact is correlated with impact velocity. This correlation provides a rate-dependent constitutive law that is potentially useful in situations such as rod penetration.

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ABSTRACT

This paper presents a method for analysing the results of a Taylor impact

test. From post mortem measurements of final specimen length and final

undeformed specimen length the dynamic material strength on impact, σ_0,

is correlated with impact velocity, V, through the relation

$$\sigma_0 = -Y - BV^2$$

where Y and B are presumed to be material constants. This relation provides

a rate-dependent constitutive law that is potentially useful in situations

such as rod penetration, for example.

INTRODUCTION

The Taylor (1948) anvil test has become one of the standard methods for assessing dynamic strengths of ductile metals. In this test a cylindrical specimen of material is accelerated to some high velocity and impacted against a relatively rigid anvil, at normal incidence. From post test measurements of the deformed shape of the specimen, the dynamic mechanical response of the test material is inferred.

Techniques for making this inference are many and varied. At their most sophisticated are finite element computer codes capable of handling very complex material constitutive relations. These utilize a great deal of post test information. For example, a fairly large number of material parameters can be adjusted to give a best fit to the entire shape of the specimen after deformation. Among the least sophisticated are simple one-dimensional analyses that assume fairly simple material constitutive relations and utilize only a small part of the post test information. For example, the original Taylor analysis (1948) calculated the strain at impact and the (rigid, perfectly plastic) material flow stress from the experimental quantities: overall post test length, and remaining undeformed length.

Both approaches have utility. The sophisticated one is required in order to give a full description of the situation. However, simple engineering theories such as that of Taylor still have considerable value. Such theories frequently provide insight into the interactions between the physical parameters, and their relationship to the outcome of the event. These interactions are, more often than not, difficult to ascertain from the previously mentioned computer analyses. As a result, simple engineering theories often provide the basis for the design of

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experiments, and are frequently used to refine the areas in which computing is to be done. It is in this spirit that the present paper is offered.

In writing formal material constitutive relations, the stress is usually assumed to be some function of strain and strain rate, less often other variables such as temperature and internal structure. In one-dimen..s..onal analyses of the Taylor test no simple estimate of strain rate presents itself when a rigid, plastic material idealization is adopted. Therefore, correlation of strength levels among tests at different impact speeds is hampered. In this paper it is proposed to correlate strength directly with velocity. This will provide a rate dependent material law that is relatively easy to extract from the Taylor test analysis. Further- more, it is in a form that could be very useful in certain situations, as for example, penetration problems.

THEORY

Consider the impact of a uniform rod upon a rigid boundary at a velocity sufficient to cause a portion of the rod to deform plastically. (The axis of the rod is assumed to lie parallel to its direction of motion and perpendicular to the surface of the target.) The initial length of the rod may be denoted as L, the initial cross-sectional area as A, and the impact velocity as V.

Let v represent the velocity with which the undeformed section of the rod is moving at time t and let ℓ represent the instantaneous length of the undeformed section. At the forward end of the undeformed section a strain discontinuity occurs. Let e denote this strain and let u denote the velocity of the rod material just within the strained region. In two earlier papers, Jones and co-workers (1987a,b) it was shown that a kinematic relationship exists

$$e\dot{\ell} = v - u \qquad (1)$$

among the variables. Here the superposed dot denotes differentiation with respect to time. These earlier papers also showed that the equation of motion of the undeformed end of the rod is

$$\ell\dot{v} + \dot{\ell}(v - u) = f(e)/\rho \qquad (2)$$

Here ρ is the density of the material (which is assumed to remain constant throughout the deformation) and $f(e) = \sigma/(1+e)$ where σ denotes true stress.

A simplified theory comparable to that of Taylor can be developed around Eqs. (1) and (2) by assuming that the material on the impact side of the rigid-plastic interface is brought instantaneously to rest. This approximation was made by Taylor and is probably relatively accurate for low velocity impacts and for the early stages of deformation during high velocity impacts. Taking u = 0, Eq. (1) becomes

$$e\dot{\ell} = v \tag{3}$$

At the same time, Eq. (2) becomes

$$d(\ell v)/dt = \sigma/\rho(1 + e) \tag{4}$$

Again following the development in Taylor's paper (1948), assume that the Eulerian plastic wave speed in the specimen is constant, say $\dot{h} = \lambda > 0$ (see Fig. 1). Since $v + \dot{\ell} + \dot{h} = 0$

$$\dot{\ell} = -(\lambda + v) \tag{5}$$

Using Eq. (5) to eliminate $\dot{\ell}$ in Eq. (3) results in

$$e = -v/(\lambda + v) \tag{6}$$

Next, eliminating the strain between Eqs. (4) and (6), applying the chain rule of differentiation and Eq. (5) gives

$$d(\ell v)/d\ell = -\sigma/\rho\lambda \tag{7}$$

Equation (7) has separable variables and can be immediately integrated.

$$\int_{V}^{v} \frac{dv}{v + \sigma/\rho\lambda} = -\int_{L}^{\ell} \frac{d\ell}{\ell} = \ell n(L/\ell) \tag{8}$$

To reduce the remaining integral in Eq. (8) requires that the functional form of σ be specified.

Earlier Jones and co-workers (1987a,b) applied the preceding theory first to the perfectly plastic material of Taylor, then to a strain hardening material. In this paper, the case of a velocity hardening material is treated. Consider a stress dependence in compression of the form

$$\sigma = -Y - Bv^2 \tag{9}$$

where Y and B are material constants. Using Eq. (9) in (8) and carrying out the indicated integration gives

$$\ell n(L/\ell) = 2\psi[\arctan(\phi\psi V - \psi) - \arctan(\phi\psi v - \psi)] \tag{10}$$

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Here $\phi = 2B/\rho\lambda$ and $\psi = [B/(\phi^2 Y - B)]^{1/2}$. Equation (10) thus leads to an explicit dependence of ℓ upon v. Let the subscript f denote final (post-impact) conditions. At the end of the event, $\ell = \ell_f$ and $v = 0$. Equation (10) then reduces to

$$\ln(L/\ell_f) = 2\psi[\arctan(\phi\psi V - \psi) - \arctan(-\psi)] \qquad (11)$$

Equation (10) provides for further manipulation of Eq. (4). Again eliminating the strain between Eqs. (4) and (6) and using Eq. (3) for $\dot{\ell}$, leads to another, first-order differential equation with separable variables.

$$\ell\dot{v} = (\lambda+v)(v - \sigma/\rho\lambda) \qquad (12)$$

Here ℓ is a specified function of v given by Eq. (10). Separating the variables and integrating gives

$$t = - \int_v^V \frac{\ell \, dv}{(\lambda+v)(v-\sigma/\rho\lambda)} \qquad (13)$$

The functional forms of σ and ℓ are specified in Eqs. (9) and (10) respectively. However, they are sufficiently complex that the integral in Eq. (13) cannot be evaluated in terms of elementary functions. But it can be evaluated *numerically*.

From numerical evaluation of the integral in Eq. (13) for $v = 0$, the final time, t_f, is obtained. This enables the final mushroom length to be calculated as $h_f = \lambda t_f$. The final specimen length can be expressed as $L_f = \ell_f + h_f$ so that

$$L_f/L - \ell_f/L = \lambda \int_0^V \frac{(\ell/L)dv}{(\lambda+v)(v-\sigma/\rho\lambda)} \qquad (14)$$

Equations (11) and (14) can now be used to calculate the plastic wave speed,

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λ, and the velocity coefficient, B.

It is reasonable to assume that for a material whose mechanical behavior is to be described by Eq. (9), the constant Y should be approximately in the range of the static yield or ultimate strengths. Consider this value to be known. Select an experimentally measured value of ℓ_f/L and an arbitrary value for λ. Then Eq. (11) can be solved for B by trial and error. After this, the right hand side of Eq. (14) can be evalauted by simple quadrature and compared with the corresponding experimentally measured $(L_f - \ell_f)/L$. According to the sign of the discrepancy a new value of λ is selected and the procedure repeated. When the correct λ is found both Eqs. (11) and (14) will be satisfied simultaneously and the values of λ and B will both be correct.

We note in passing that these results are intermediate in complexity between the two earlier analyses. For the simple rigid, plastic material, Jones, et al. (1987a), both the equations for ℓ and t could be integrated explicitly. For the work hardening material, Gillis, et al. (1987b), neither could be. For the velocity dependent material suggested here, the equation for undeformed length can be explicitly integrated but numerical integration is required in the time equation.

RESULTS AND DISCUSSION

Taylor test specimens were taken from a copper rod. These were all of diameter 7.62mm (0.300 in) and had length-to-diameter ratios ranging from 1 to 10. The mechanical response of this material was depicted earlier, Gillis, et al. (1987b), but for the present purpose it can be noted that the static yield strength was about 230 MPa and the ultimate tensile strength about 300 MPa. The average of these two values was taken as Y in the calculations.

Table 1 shows the results of a dozen Taylor impact tests. These results are arranged in order of increasing initial velocity within each set of specimens having the same aspect ratio. With the exception of those specimens having aspect ratios of 2 or 1 the results are quite consistent.

The calculated values for plastic wave speed are remarkably constant. However, they do tend to decrease slightly within each aspect ratio with increasing impact velocity. This same trend is observed in the velocity coefficients except for the two tests at an aspect ratio of three. The dynamic strengths on impact are all fairly close to those obtained in the two previous analyses, Jones and co-workers (1987a,b).

Changing the value of Y to 230 MPa generally results in somewhat lower calculated plastic wave speeds and somewhat higher values for both the velocity coefficients and the dynamic strength on impact. The opposite occurs taking Y as 300 MPa.

Those previous analyses illustrated how the underlying scheme of the present work could be applied to simple non-hardening, and hardening materials. In the present paper, application is extended to an explicitly rate-sensitive material. In this formulation, rigid rod velocity was selected as the best variable to describe the deformation rate. This choice

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was largely one of convenience.

As noted in the two earlier papers the principal shortcoming of the present procedure is that only two equations exist. These can be used to determine only two unknowns. One of the unknowns has to be the plastic wave speed so only one parameter can then be determined for a material constitutive law.

CONCLUSIONS

A method was presented for the analysis of the Taylor impact test which accommodates the correlation of dynamic strength with deformation velocity.

This method was illustrated using the velocity hardening material description given above as Equation (9).

This method is limited to finding the value of only one parameter in the material description.

ACKNOWLEDGMENT

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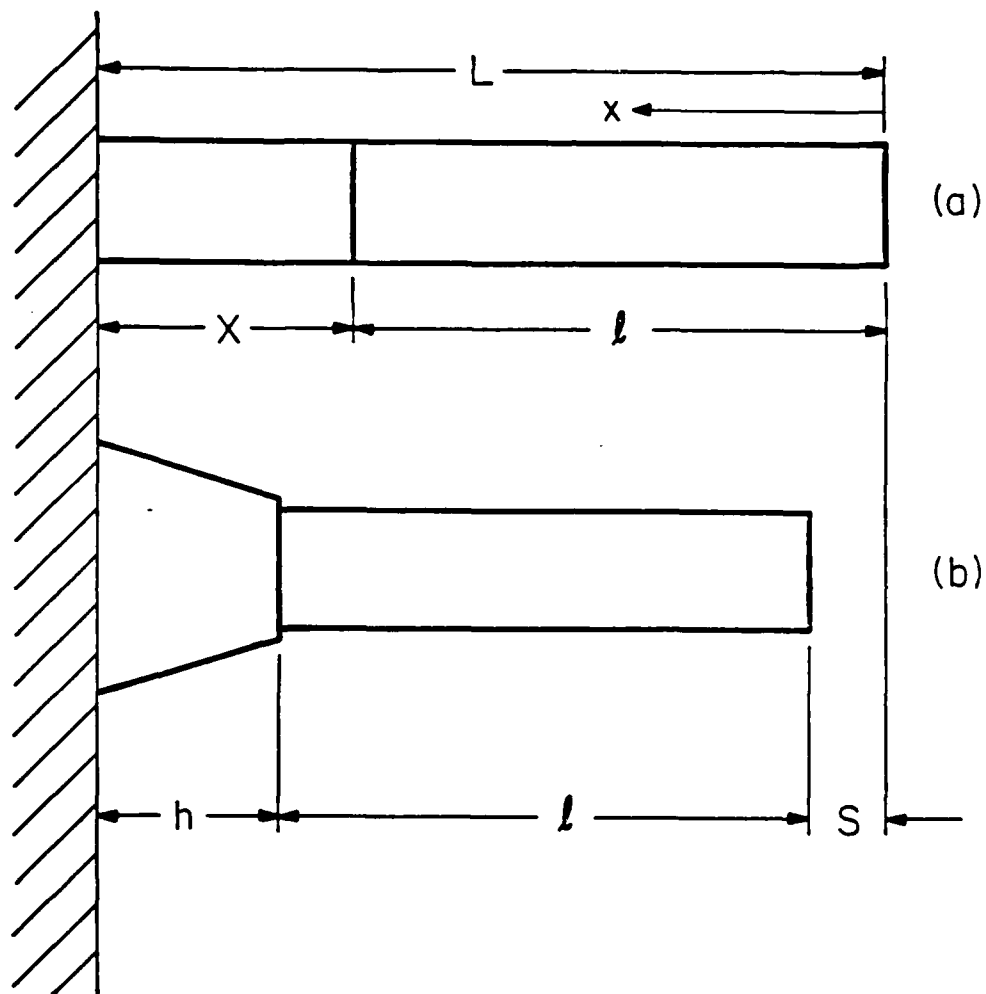
Figure 1. Schematic illustration of projectile impacting a rigid
anvil. The notation used in the paper is indicated in
the drawing. The upper view shows the reference con-
figuration of the projectile and the lower view shows
its deformed configuration.

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TABLE 1. TAYLOR IMPACT TEST RESULTS

Impact Velocity	Aspect Ratio	Final Length	Final Undeformed Length	Plastic Wave Speed	Hardening Coefficient	Dynamic Strength on Impact
V	L/D	L_f/L	ℓ_f/L	λ	B	σ_0
m/s				m/s	$kPa/(m/s)^2$	MPa
83	10	0.924	0.416	295	42.8	560
95	10	0.906	0.417	259	33.7	569
96	10	0.906	0.418	259	34.4	582
94	5	0.909	0.419	264	36.2	585
99	5	0.900	0.414	250	31.2	571
104	5	0.894	0.400	251	31.2	602
152	5	0.793	0.250	224	14.3	595
142	3	0.817	0.261	239	17.5	613
153	3	0.826	0.311	227	22.0	780
130	2	0.865	0.343	251	29.9	770
157	2	0.825	0.351	206	21.0	783
151	1	0.849	0.235	307	35.2	1068

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